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A Math Textbook Time Machine for teacher training

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Abstract. The complexity of mathematics textbooks' layout has been increasing due to technical advances since the 1960s. Whereas teachers used to be the only authors of textbooks, layout and graphic designers now play a central role in the transmission of knowledge. Design choices in a layout may however confuse learners if the general, didactic, and mathematical meanings interfere. We conducted a semiotic analysis of six textbooks from the 1960s to the 2010s, and built an interactive web-based visualization tool around our observations. The MTTM tool (Math Textbook Time Machine) displays the same mathematics lesson in the design style of six different decades, and allows a user to explore them as if travelling through time. We propose possible uses of this tool in teacher education to raise awareness of the possible confusion induced by design choices.

Keywords: semiotics, math textbooks, historical awareness, design, technology-enhanced teaching.

1. Introduction

The aim of this article is to describe the design of a web-based tool for promoting teachers' historical awareness of the changes that have occurred in textbooks and other teaching material as a result of a technical revolution. The look and feel of textbooks, both paper and computer-based, has indeed tremendously changed during the past six decades. Historical awareness of the ways in which textbooks, as tools of the teacher trade, have evolved could help teachers to identify potential obstacles in learning which may occur as a result of modern text and graphic design and newly designed visualizations. The identification of such *semiotic*, rather than epistemic, obstacles might be of crucial importance in mathematics learning because of its heavy dependence on external representations. According to Duval

(2006), mathematics involve multiple semiotic registers for reasoning about and for operating upon mathematical objects because the latter are themselves, by nature, unavailable to the senses.

The first section of this article describes the theoretical frame which focuses on the distinctions between teachers and designers and between domain-specific and generic (all-purpose) text and graphics. The second section presents a semiotic analysis of mathematics textbooks throughout recent history via a sample of six textbooks. The third section explains the design of the web-based tool called the Math Textbook Time Machine (MTTM), and its implementation in web format through separation of content and style sheets. The final section of this paper contains recommendations for promoting teachers' historical awareness through teacher training using the MTTM.

2. Theoretical frame

Texts and graphics in textbooks can be seen as the modern equivalent of the shadows on the wall of Plato's cave, as proposed by de Vries (2012). In this analogy, the learners map onto the prisoners in the cave, and the teachers map onto the puppet showmen who carry objects above their heads in order to produce the shadows on the wall of the cave. An important issue in Plato's allegory concerns whether or not the study of the shadows on the wall will allow prisoners to construct knowledge of the objects in the real world. Nowadays, this issue is still studied by researchers such as Mayer (2009), through his cognitive theory of multimedia learning, and Schnotz (2002). Another issue in Plato's cave concerns the puppet showmen's choice of objects to carry above their heads for producing appropriate shadows. This issue has received much less attention in the literature, except for Ainsworth's (2006) DeFT (Design, Functions, Tasks) framework for learning with multiple representations. The construction of the theoretical frame builds on the cave analogy in order to exemplify three critical ways in which today's situation differs from the ancient one: 1) a shift from teachers to designers in charge of teaching material, 2) changes in the nature of the represented world, 3) the importance of domain-specific representational formats.

2.1 The technical revolution and the designer-teacher gap

In the last decades, a technical revolution of textbooks has taken place. It is characterized by the emergence of new objects that extend the definition of a textbook: CD-ROM, DVD, internet sites and other platforms, and e-textbooks. The educational use of these new technologies has been extensively studied (e.g., Heid & Blume, 2008). The exploration of the potential for teaching and learning might well lead to the generalization of new uses in the classroom of the future.

However, traditional paper textbooks continue to be massively edited and used. One could even argue that the technical revolution is only an illusion. Indeed, only a minority of contemporary textbooks are e-textbooks. Also, a variety of teaching media such as video, audio, or posters, already exist and have been used in the classroom for a long time (Choppin, 2008). Finally, some e-textbooks are mere replicas of paper textbooks in an electronic format. In consequence, whether or not textbooks are undergoing a technical revolution will only be told by future use and choice of pedagogical materials. Even if textbooks are actually undergoing an evolution rather than a revolution, the transformations they went through in the last decades might be of central importance to their good use in the future.

Another way to look at this is to consider how traditional paper textbooks evolved during the last decades as a result of the same technical revolution. Of course, culture, society, and trends in psychological theories also influence the design of textbooks (Ellis & Berry III, 2005), but technology nonetheless clearly affects the production system: publisher strategies, designer tools, print possibilities (Tarbouriech & Bruillard, 2005). Publishers have to conciliate the emergence of new teaching media with the existing textbook market. But sometimes the strategy simply seems to focus on proposing textbooks which look up to date and modish (Moeglin, 2010). Designers are in charge of this desired attractiveness through the layout and formatting. A major increase of available graphic design tools and print possibilities (color choice, quality) indeed allows for fancy textbooks. Borne (1998) even concluded that the main weakness of modern textbooks lies in the excess of materials rather than in the lack of them.

Bezemer and Kress (2008, 2010) observed the evolution of textbooks in secondary school in England (1930s, 1980s, and 2000s). An analysis of the use of typography, image, writing, and layout showed a modified role for layout. The survey shows how layout has become a major resource for constructing the learning environment. Such changes in design require new forms of literacy: “fluency not only in ‘reading’ writing, image, typography and layout jointly, but in the overall design of learning environments” (Bezemer & Kress, 2010). Bezemer and Kress (2010) describe a transition of the responsibility for coherence from authors to designers. Through the manipulation of layout, designers can create cohesion in a way the authors cannot. The Borne report (1998) observes the same gap between designers and authors in French textbooks:

> It is exceptional to encounter a well-built and coherent passage favorable to simple continuous reading. The effect of typography, bolded characters, italics, highlighting, boxes, and references to other pages, to documents, or to images forces the reader’s attention to hop around and move back and forth on the page in an inappropriate way for reading. (p.14, our translation)

The observed shift from teachers to textbook designers is an instance of a more general phenomenon, namely the intervention of designers between a user’s need (to cook, to teach, etc.) and the construction of an artefact to satisfy that need (an oven, a textbook). In modern society, in contrast to ancient times, hardly anyone manufactures his or her own objects and tools. In terms of the cave analogy, the puppet showmen are no longer alone in deciding which objects to select and how to carry them above their heads. The consequences of such a separation between designer and user, in terms of error proneness, have been widely documented, for example by Norman (2013). Layout and formatting by graphic designers may actually induce obstacles for learners. One approach to the issue would be to involve teachers in the design process. Some researchers, such as Even and Ayalon (2014) or Gueudet, Pepin, Sabra and Trouche (2014), have started to document the approach. Alternatively, Gravemeijer (2014) suggests that textbook design should be introduced in teacher professionalization. The approach adopted in this article proposes an actual tool for accomplishing this introduction in teacher training, namely through promoting teacher trainees’ historical awareness of the potential problems which arise from the use of modern graphic design in textbooks.

2.2 Contemporary text and graphics for learning

Although the combination of text and pictures for learning is as old as Comenius's *Orbis Pictus* (Visible World in Pictures), today's teaching material contains an enormous variety of inscriptions, representational modes, and styles. The contemporary textbook differs from the ancient situation in at least three ways. First, both in Plato's cave and in Comenius's illustrated textbook, graphics serve the purpose of representing the creatures, objects, and visible phenomena that surround learners and observers. Today, however, teaching material also needs to display largely invisible and abstract objects and phenomena, such as chemical processes, laws of nature, and mathematical objects like functions, polynomials, or probabilities.

Second, whereas ancient visualizations relied on resemblance relations, known as iconicity, modern graphics now involve symbolic representations that depend on conventions, that is, shared arbitrary relations. In educational psychology, pictures are often categorized as depictive (iconic) representations and text as a descriptive (symbolic) representation. However, this only holds in denotation, when the literal meaning of a word or picture is concerned. For instance, the word "lightbulb" in an instructional text uniquely refers to a device producing light from electricity. But a picture of a lightbulb, through connotation, might also mean a new idea. In fact, many words, as well as their depictions, can have figurative meaning (connotation). Examples in science education are numerous; consider for instance "work", "energy", "lever", or "heart". Even Barthes' (1964) third mode, metalanguage, is relied upon today in domain specific schemas. For example, in the graphical language of electrical circuit diagrams, a particular graphical element, rather than an actual lightbulb, represents a lightbulb. In sum, contemporary text and graphics signify in many different ways.

And third, teaching material nowadays also contains inscriptions (text, symbols, graphics, equations, and animations) fulfilling other functions than mere representation of the content. For example, the content needs to be structured through headers and subheaders and presented from more basic to more complex. It also may be necessary to comment or to otherwise guide the learner through the textbook. Finally, textbooks nowadays also contain inscriptions for the purpose of making them more attractive.

2.3 The importance of domain-specificity

The gap between designers and authors raises the issue of the importance of domain-specificity of visualizations and more generally of any form of external representation. Didactical text inherently contains domain-specific terms which need to be introduced by giving definitions which themselves may contain domain-specific terms. For example, a definition of "pollination" would contain words like "anther", "pollen", and "stigma", a definition of "mammal" would contain words like "clade" and "mammary gland". As mentioned above, instructional texts most of the time rely on denotation and will not mix different meanings of the same word. A physics textbook, for example, will probably not contain a sentence like: "In his work, a carpenter does positive work when he uses a hammer to drive a nail into a workpiece". The same is true for visualizations (see Figure 1). A straightforward internet image search with the keywords "aqua", "water" or "H2O structure" gives an idea of the myriad of ways to represent water molecules graphically. These visualizations build on everyday objects (balls and sticks in a ball-and-stick model), letters and digits (H, O, 2, 104.5), and graphical elements (lines, dots, arrows, circles). None of these visualizations can actually be interpreted according to pure resemblance relations. Such domain-specific formats of representation are qualified as monosemic in Bertin's (1983) terms, because each graphical element has a unique meaning which is attributed regardless of context, prior to understanding the configuration in which it is inserted.

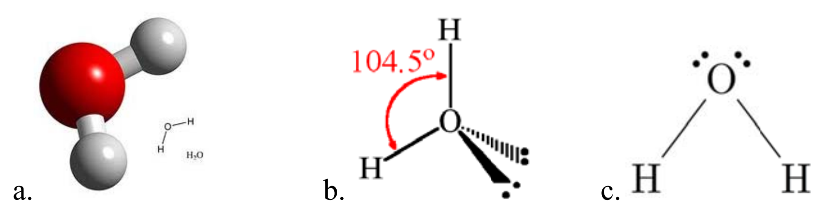


Figure 1. Domain-specific representations of water (H_2O).

In contrast, without any knowledge of the domain or of the domain-specific format, it is difficult to determine which graphical elements should be strictly interpreted according to the domain-specific meaning, and which graphical elements are independent from this format and should be interpreted according to the general meaning. For example in Figure 1b, the lines connecting the letters, as prescribed by the domain-specific format, specifically represent chemical bonds and not any other kind of relationship. The bidirectional arrow, however, does not pertain to the domain-specific format and was added as a comment to specify the angle magnitude (see also “Adding a comment or an alert” in the next Section). It might also be difficult to know whether to interpret a given graphical element according to resemblance relations or according to convention, and furthermore whether the author intended denotative or connotative meaning. For example, in Figure 1, the relative position of the atoms should be interpreted according to resemblance. However, the letters used (e.g., O for Oxygen) or the color of the balls should be interpreted according to convention. These potential sources of confusion exist for a learner because learning involves getting to know both the content and the different domain-specific ways of representing that content. Such visualizations are however not confusing for teachers. Knowing the domain-specific representational formats prevents a teacher from mixing up different types of representations. More importantly, the relative *transparence* for the teacher is likely to *conceal* potential sources of confusion and semiotic obstacles for their learners. For example, in reference to the bidirectional arrow in Figure 1, a teacher might not feel the need to explain that the arrow is commenting rather than representing (i.e., the arrow is not part of the domain-specific format). The MTTM was specifically designed to raise teachers’ awareness of such non-mathematical obstacles in mathematics learning.

3. Semiotic analysis of math textbooks

In order to build the web-based tool, first a semiotic analysis of math books was carried out. The analysis method involved examining how different functions were accomplished for an identical lesson throughout several decades. Based on Carney & Levin’s (2002) typology of the functions of pictures in instructional material, we identified five functions of any inscription, textual or graphical, in mathematics textbooks. The first function is structuring the content (organizational function). This is a more general function accomplished by a table of contents, headers, etc. in any book. The second function is representing the content (representational function). As has been mentioned in the introduction, this is a crucial function in the case of mathematics since mathematical objects are not in any way visible of themselves. The third function is adding a comment or an alert. In fact, in addition to text and graphics, a textbook may also contain floating text or graphics for drawing the learner's attention to particular issues. The fourth function is to embellish (decorative function) which is important for a publisher in selling a textbook. Finally, the fifth function is to allow the learner to interact with the textbook. Although this function might be more important in e-textbooks, some examples may be found in paper textbooks.

The next step involved choosing the actual corpus for analysis. In order to find comparable materials, we looked for textbooks at identical school level and containing the same lesson. It was also required that the lesson contained various didactic elements such as definitions, properties, examples, equations, tables, and graphs. By exploring a variety of textbooks, we found that the lesson on polynomial functions of degree 2 was present in all decades. Also, this lesson was always taught in the junior year of high school, the French "*Seconde*". This lesson contained the required variety of didactic elements and was kept as analysis material.

Furthermore, the choice of textbooks was made according to the availability of complete collections in our sample. We needed one textbook for each of six decades, from the 1960s to the 2010s. We selected five textbooks from the publisher *Hachette*, with only the 1970s textbook being taken from the publisher *Bordas*. The semiotic analysis therefore consists of investigating all five functions for the same mathematics lesson in six textbooks representative of six different decades.

3.1 Structuring the content

First, in each textbook's version of the lesson, we identified the layout elements for structuring the content. We characterized these elements independently from their didactic meaning. We identified the following:

- page body,
- titles,
- table of contents,
- various levels of headings,
- paragraphs,
- lists,

- notes,
- tables,
- graphs,
- footers,
- page numbers,
- pictures,
- other graphics, such as arrows.

3.2 Representing mathematical objects

Then, we identified the elements in terms of their didactic meaning, regardless of their place in the layout. These elements, specific to the domain of mathematics, were:

- definitions,
- properties,
- examples,
- remarks,
- consequences,
- demonstrations,
- mathematical operators,
- equations,
- tables of variations,
- line graphs,
- comments,
- cross-references.

The two analyses of structuring and representing were done recursively because various sub-elements could be found inside of a parent element, such as notes inside of a paragraph or equations inside of a definition. In fact, it should be noted that the two types of elements actually interfere with one another: all contents both have a place in the layout as well as a didactic meaning regarding mathematics. Thus, the elements found in the two analyses were considered together and intersected to each other, in order to take simultaneously into consideration both the place in the layout and the didactic meaning related to mathematics. This created a long list of composite elements different from one another. For instance, ‘property paragraphs’ could differ from ‘example paragraphs’, and ‘property headings’ could differ from ‘property paragraphs’.

Propriété admise

Dans un repère orthogonal, la parabole représentant une fonction polynôme de degré 2 admet pour axe de symétrie la **droite parallèle** à l'**axe des ordonnées** passant par le **sommet**.

Définition

On appelle **fonction homographique** toute fonction de la forme **x › (ax + b) / (cx + d)** où a, b, c et d sont des réels fixés avec c ≠ 0 et a × d ≠ b × c. La courbe représentative s'appelle une **hyperbole**.

Figure 2. Prevalence of structure over mathematics and inconsistent use of text color (2000 style sheet)

Figure 2 shows an example of this intertwining of the two. Note the two subheadings which present identical layout. An alternative way would have been to choose a different layout for different mathematical objects, that is, a header layout for properties and a different one for definitions. In sum, the semiotic analysis showed a prevalence of structure over mathematical content.

From each composite element of each version of the lesson, we observed a set of layout features which are actually in charge of both functions. These features are listed in Table 1 along with examples of some possible values found in the set of textbooks. The table also indicates the range of the values found, that is both the minimum and the maximum of the number of different values found in one lesson version. The variation in some features, like the number of different fonts, did not seem to change much between the decades. However, some features like the use of color seemed to clearly increase over time.

However, each feature can lead to clarity or confusion according to how it interacts with other features of other elements. For instance, we observed that more complexity in the layout did not seem to allow identifying and differentiating more elements of the lesson. The lesson elements were the same, but each had a greater number of specific features. In this case, the complexity does not benefit didactical purpose. In a few other cases, elements were differentiated from one another, but without a clear purpose.

This was the case with the use of colored text in the 2000s lesson. Words were written in purple, green, blue, or red without an apparent didactic motive for the color choice (Figure 2). Such inconsistencies in the use of color may possibly lead to confusion for the learner.

Table 1. Layout features, value examples, and range of variation

Features	**Value examples**	**Minimum**	**Maximum**
Placement	35 mm margin Centrally aligned	32 *1960, 1970*	48 *1980*
Size	120 mm Half of page	13 *1960, 1970*	35 *2000*
Color	Blue Orange gradient background	1 *1960, 1970*	10 *2000*
Borders	Thick Red Double	0 *1970, 2010*	6 *2000*
Font	Serif 12 points Bolded Italics Uppercase	17 *1990*	34 *2000*

3.3 Adding a comment or an alert

The most recent textbook (2010) showed extra-textual insertions for adding a comment or an alert. Figure 3d shows an alert attached to an arrow and a lightbulb. Note how an arrow can have a number of significations and therefore requires further analysis. Looking at arrow-like signs, we can observe the coexistence of arrows in the mathematical symbolism as well as in a general use, non-specific of the domain (see Figure 3 for examples):

- Mathematical: mathematical object (i.e., a vector; no occurrences in the lessons on polynomial functions),
- Mathematical: transformation of x in $f(x)$ as in Figure 3a,
- Mathematical: axis in a coordinate system as in Figure 3c,
- Mathematical: in a variation table, for the ascending or descending part of a function, as in Figure 3b,
- General: deictic, give the name of a mathematical object, as in Figure 3c,
- General: deictic, insert a comment, as in Figure 3d,

- General: deictic, cross-reference, like the triangle (arrow without a tail) as in figure 3d and Figure 5.

a. ***Définition*** On appelle **fonction polynôme de degré 2** toute fonction de la forme **x → ax² + bx + c**

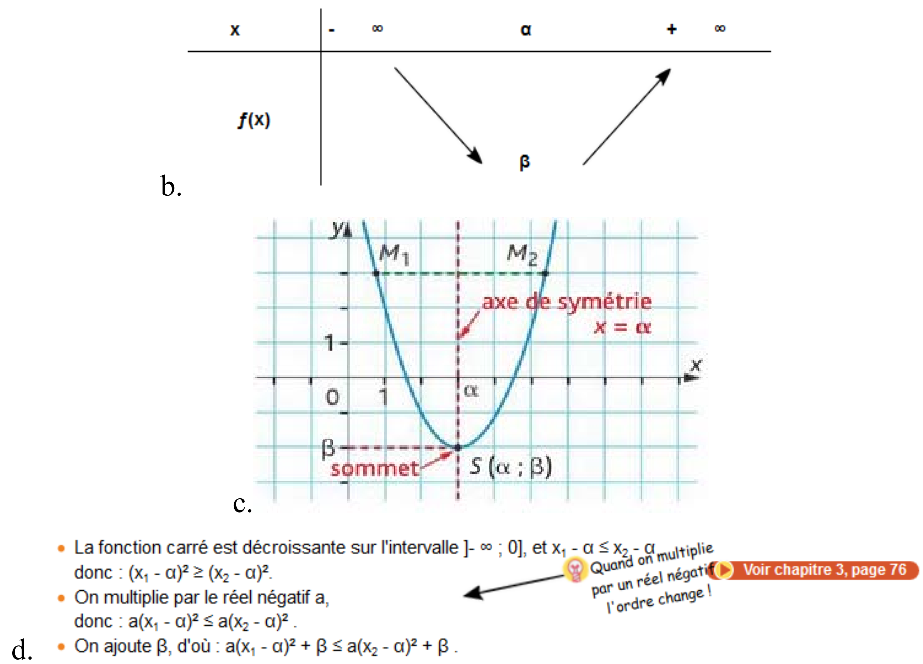


Figure 3. Different uses of arrows. a) Transform in 1970 style sheet, b) variation table in 1960 style sheet, c) axis and label in 2010 style sheet, d) comment and refer in 2010 style sheet.

3.4 Embellishing

More recent textbooks showed decorations by means of pictures and graphics. Figure 4 shows an example of a decoration. A lower-case sigma seems to be used in this case as a typographical artifice to signal a chapter heading for a lesson. This could be understood as the result of a designer's desire to symbolize mathematics, as mathematics is the domain in the textbook at hand. In introducing such a symbol, the designer did not take into account that a sigma has a specific meaning, for example in statistics where σ stands for population standard deviation, or in physics where it stands for electric conductivity. Again, such decorative use might be confusing for the learner, especially in this case where the sigma contains the chapter number and as such violating the domain specific conventions for using a sigma.

Figure 4. Decorative use of a sigma in a chapter heading in 1980 style sheet

3.5 Interacting

The fifth function would be particularly relevant for electronic textbooks. Just like any computer program, in contrast to a book, an electronic textbook involves an interface. The screen is not only used as the page of a book, but it also relies on inscriptions for operating it. Examples of operations for an e-book would be opening and closing a book, turning pages, crisscrossing the book by clicking on hotwords and hyperlinks. Nowadays, textbooks may contain a paper equivalent of a hyperlink. We did find such cross-referencing in the two most recent textbooks (see Figure 3d and Figure 5 for examples). In Figure 5, a small red triangle is used. This might be confusing to the extent that mathematics textbooks also contain chapters on geometry in which a triangle represents an instance of the mathematical object 'triangle'.

L'écriture $a(x - \alpha)^2 + \beta$ d'une fonction polynôme de degré 2 s'appelle sa **forme canonique** ; on peut l'obtenir avec un logiciel de calcul formel.

► *Voir Découvrir 2, page 96*

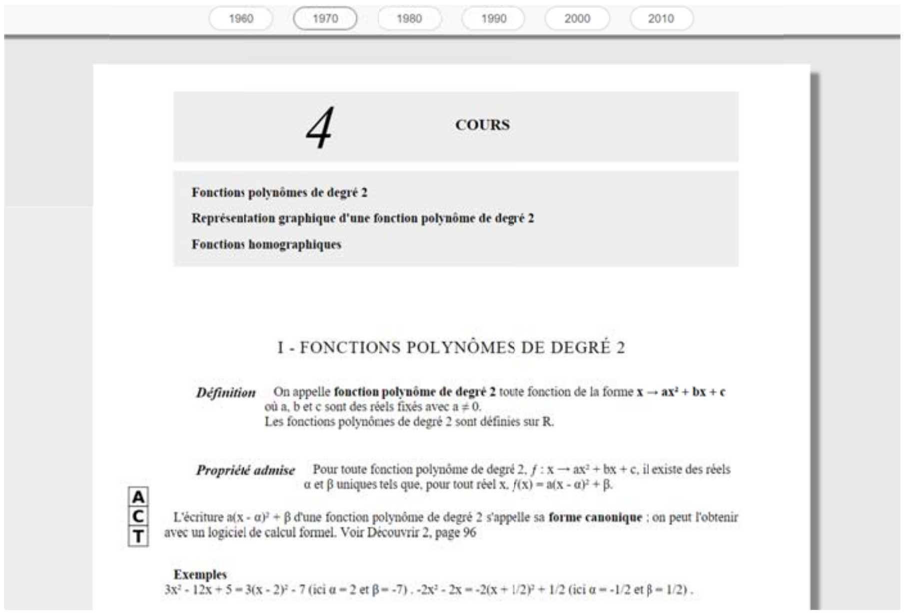
Figure 5. A reference; the paper equivalent of a hyperlink, in 2000 style sheet.

4. Design of the Math Textbook Time Machine

In order to visualize the results of our semiotic analysis, we created a computer-based tool integrating six different graphical layouts, one for each lesson variation, displayable on demand onto unchanging textual contents. The textual contents were extracted from the most recent textbook, in order to be most relevant to the contemporary reader. The textual contents were then coded into an HTML file along with the markup corresponding to all the generic and domain-specific elements uncovered in the analysis. Then, one CSS style file was created for each lesson variation. Each CSS file linked each generic or domain-specific element to the decade-specific features identified in the semiotic analysis. Consequently, when a certain CSS file was loaded, the contents were displayed in the style of the corresponding decade. Finally, a navigation bar containing one button per decade was added to the program, loading the different layouts in a single mouse click and allowing easy navigation between decades (Figure 6; interactive tool available at https://db.tt/FUzIqsN4).

Users of the MTTM simply need to click on the time buttons to see the layout of the page change accordingly. Thus, out of the box, the MTTM allows observations and comparisons of successively displayed layouts. As we will discuss in section 5 of this paper, these observations and comparisons could be the focus of specific training tasks. Also, because it is implemented in open web formats (HTML and CSS), adapting the MTTM only requires some knowledge of the coding languages at hand. For instance, the lesson contents could be changed for a specific training or study. Another possibility is to add other layouts, in order to visualize historical layouts, to try to time travel to the future of mathematics textbooks, or to test and compare experimental layouts.

The MTTM can thus be adapted into a design, research, or training tool. In the last section of this paper, we focus on its capabilities as a teacher training tool, the use for which it was originally built.



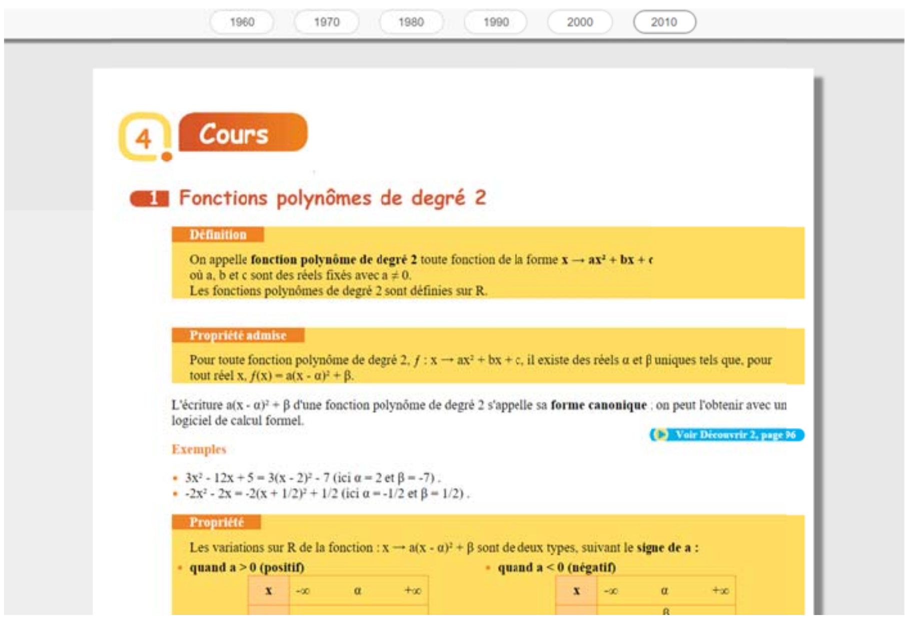


Figure 6. Screenshots of the MTTM, decades 1970s and 2010s

5. Recommendations for teacher training

As previously said, the analysis of textbook design evolution highlights a shift in authority from authors to designers. In their work, designers do not alter textual contents of mathematics lessons. However, they may change the format of data tables, modify the colors of graph axes, wrap different definitions inside a box, or add warning symbols in the margin. By these means, they transform the way mathematical objects are to be apprehended. They play a part of the teacher's role by emphasizing, categorizing, and pointing at contents, thus operating in the didactical domain.

Consequently, it can be argued that teachers should be aware of the interferences between design decisions and didactic goals, and ideally participate themselves in the design of textbooks, as suggested by Even and Ayalon (2014). Furthermore, this awareness must be a goal in teacher training, because their domain expertise prevents teachers from experiencing the designer-induced potential confusions *themselves* and thus makes them ineffective in identifying these semiotic obstacles for their learners.

Simply discovering the MTTM can be the starting point of a general discussion about these interferences between a trainer and trainees. Furthermore, more specific tasks may raise trainees' awareness of the function of inscriptions or of the possible confusion between domain-specific and common graphical elements.

In order to explore the function of signs, a trainee may be asked to search for indications of emphasis (bold, color, underlining, frame, etc.). The trainee would then comment on the evolution of specific layout features or discuss with the trainer what they think is important. Trainees could also evaluate the coherence of text and graphics in the MTTM. The designer approach of the categorization of objects is to apply the same design to objects of the same nature. Identical design applied to titles or subtitles at the same level does not contradict mathematics, but applying identical design to both mathematical definitions and properties may actually do so (e.g., whereas a definition has no proof, a property may have one). Thus, trainees may search for categorizations induced by formatting and assess whether they contradict mathematical reasoning.

The deictic function (i.e., pointing) is mainly supported by arrows (see "Adding a comment or an alert" above). Arrows constitute an excellent opportunity for trainees to realize the risk of confusion between representations (mathematical signs) and comments (domain-non-specific signs) and could be a first focus point in a trainee's exploration of the MTTM. In mathematics, arrows do not point at objects but are full-blown representations in themselves. For instance in a variation table, upward-pointing arrows define that the function is increasing in this interval. Likewise, vector arrows on a graph carry their meaning through their length and direction, not through the object they point at. However, in general use, arrows have a clear deictic function such as commenting (pointing at a difficulty), or naming (pointing at an object). Arrows used in cross-references are also deictic, but point at an external object. Being aware of this polysemy may allow for a better understanding of some student difficulties.

Arrows and similar graphical elements may also be used to highlight the distinction between similarity and arbitrary relations. An arrow in a variation table displays a similarity relation with its mathematical meaning (increasing or decreasing), whereas most arrows have an arbitrary, symbolic relation to their meaning. Despite its many monosemic graphics, mathematical textbooks therefore contain a form of polysemy (Bertin, 1983) which requires interpreting each sign depending on its immediate context. In manipulating the MTTM, trainees would be able to recognize the larger complexity in the more recent textbooks leading to more polysemy.

6. Conclusion

The future of classroom mathematics education is yet to be told, but textbooks are most probably going to be a central part of it, in one form or another. As more technical advances are used in the design of textbooks, more distance separates the textbook author from the final object in the hands of students. In paper textbooks, as we studied, designers may interfere with the didactic and mathematical meaning of both text and graphics. In electronic textbooks, maybe programmers or other professionals on the production line will play such an accidental didactical role. Teachers should be aware of the possible consequences for their students, and take them into account when teaching or choosing the textbooks they want to work with. By visualizing the evolution of design in the past six decades, the MTTM can raise trainee teachers' awareness and lead them to a more critical use and choice of textbooks.

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Textbooks

Beltramone, J. P., Brun, V., Labrosse, J., Merdy, C., Misset, L., Rousseau, P., Sidokpohou, O., Talamoni, C., & Truchan, A. (2010). *Déclic Mathématiques 2e*. Paris: Hachette.

Coassart, E., Théron, P., Théron, P., & Mordelet, C. (1970). *Collection de Mathématiques 2e A, C, T, Tome II*. France: Bordas.

Gautier, C., Gerll, D., Thiercé, C., & Warusfel, A. (1981). *Mathématique 2e*. Paris: Hachette.

Gautier, C., Thiercé, C., Gerll, D., Sanchez, J. P., & Jozeau, M. F. (1990). *Mathématiques 2e: Collection Perspectives*. Paris: Hachette.

Maillard, R., & Millet, A. (1961). *Mathématiques 2e A'CMM'*. Paris: Hachette.

Misset, L., Coste, R., Guerlou, C., Lotz, E., & Turner, J. (2000). *Déclic Maths Seconde*. Paris: Hachette.

References

Ainsworth, S. (2006). DeFT: A conceptual framework for considering learning with multiple representations. *Learning and Instruction*, *16*(3), 183–198. http://doi.org/10.1016/j.learninstruc.2006.03.001

Barthes, R. (1964). Éléments de sémiologie. *Communications*, *4*(1), 91–135.

Bertin, J. (1983). *Semiology of graphics: Diagrams, networks, maps (WJ Berg, Trans.)*. Madison, WI: The University of Wisconsin Press, Ltd.

Bezemer, J., & Kress, G. (2008). Writing in multimodal texts a social semiotic account of designs for learning. *Written Communication*, *25*(2), 166–195.

Bezemer, J., & Kress, G. (2010). Changing text: A social semiotic analysis of textbooks. *Designs for Learning*, *3*(1-2), 10–29.

Borne, D. (1998). *Le manuel scolaire* (p. 47). Paris: IGEN: La documentation Française: Le Manuel scolaire. Ministère de l'éducation nationale, de la recherche et de la technologie.

Carney, R. N., & Levin, J. R. (2002). Pictorial illustrations still improve students' learning from text. *Educational Psychology Review*, *14*(1), 5–26.

Choppin, A. (2008). Le manuel scolaire, une fausse évidence historique. *Histoire de l'Éducation*, *117*, 7–56.

De Vries, E. (2012). Learning with external representations. In N. M. Seel (Ed.), *Encyclopedia of the Sciences of Learning* (pp. 2016–2019). Heidelberg: Springer.

Duval, R. (2006). A cognitive analysis of problems of comprehension in a learning of mathematics. *Educational Studies in Mathematics*, *61*(1-2), 103–131.

Ellis, M. W., & Berry III, R. Q. (2005). The Paradigm Shift in Mathematics Education: Explanations and Implications of Reforming Conceptions of Teaching and Learning. *Mathematics Educator*, *15*(1), 7–17.

Even, R., & Ayalon, M. (2014). Teachers editing textbooks: Transforming conventional connections among teachers, curriculum developers, mathematicians, and researchers. In *Conference on Mathematics Textbook Research and Development (ICMT-2014)* (p. 37).

Gravemeijer, K. P. (2014). Transforming mathematics education: The role of textbooks and teachers. In Y. Li, E. A. Silver, & S. Li (Eds.), *Transforming mathematics instruction* (pp. 153–172). Springer.

Gueudet, G., Pepin, B., Sabra, H., & Trouche, L. (2014). The design of and interaction with e-textbooks: a collective teacher engagement. In *International Conference on Mathematics Textbook Research and Development (ICMT-2014)* (pp. 239–244). University of Southampton.

Heid, M. K., & Blume, G. W. (2008). *Research on Technology and the Teaching and Learning of Mathematics: Research syntheses* (Vol. 1). IAP.

Mayer, R. E. (2009). *Multimedia learning*. Cambridge University Press.

Moeglin, P. (2010). *Les industries éducatives*. Paris: Presses Universitaires de France.

Norman, D. A. (2013). *The design of everyday things: Revised and expanded edition*. Basic books.

Schnotz, W. (2002). Towards an integrated view of learning from text and visual displays. *Educational Psychology Review*, *14*(1), 101–120.

Tarbouriech, D., & Bruillard, É. (2005). Study of mathematics textbooks at primary and low secondary level. In M. Hornsley, S. V. Knudsen & S. Selander (Eds.), *Has past passed? Textbooks and educational media for the 21st Century* (pp. 213–221). HLS..